Pipelined Consensus for Global State Estimation in Multi-Agent Systems

Golnaz Habibi
Rice University
golnaz.habibi@gmail.com

Zachary Kingston
Rice University
zkk1@rice.edu

Zijian Wang
Boston University
zjwang@bu.edu

Mac Schwager
Boston University
schwager@bu.edu

James McLurkin
Rice University
jmclurkin@rice.edu

ABSTRACT

This paper presents *pipelined consensus*, an extension of pair-wise gossip-based consensus, for multi-agent systems using mesh networks. Each agent starts a new consensus in each round of gossiping, and stores the intermediate results for the previous k consensus in a *pipeline* message. After k rounds of gossiping, the results of the first consensus are ready. The pipeline keeps each consensus independent, so any errors only persist for k rounds. This makes pipelined consensus robust to many real-world problems that other algorithms cannot handle, including message loss, changes in network topology, sensor variance, and changes in agent population. The algorithm is fully distributed and self-stabilizing, and uses a communication message of fixed size. We demonstrate the efficiency of pipelined consensus in two scenarios: computing mean sensor values in a distributed sensor network, and computing a centroid estimate in a multi-robot system. We provide extensive simulation results, and real-world experiments with up to 24 agents. The algorithm produces accurate results, and handles all of the disturbances mentioned above.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems

General Terms

Algorithms, Design, Experimentation

Keywords

Distributed, Consensus, Communication Failure, Centroid Estimation, Multi-Robot

1. INTRODUCTION

Constructing local estimates of global state is a critical utility in multi-agent systems. For example, in an environment monitoring scenario where a group of mobile agents are deployed to cover a large area [19], we may be curious about measuring the average value gathered across all the agents to decide if something unusual or dangerous is happening. In a social network of human agents [6], we are interested in how global ideas and opinions form, spread, and cluster. Knowing the global state information can help agents coordinate more efficiently. Flocks of birds, schools of fish, and crowds of people measure the motion of the nearby neighbors in order to achieve large group-level formations [23]. In a network of mobile agents, sharing local measurement probability information can be used to produce a more informative configuration of the positions of the nodes [9]. In the work on cooperative transportation by a group of robotic agents [24], inferring the forces from other agents let each individual agent know how to align its own force with others' to achieve efficient cooperation.

These examples all require global state estimates in distributed multi-agent systems. Inferring global information from local information is a fundamental theme is multi-agent systems. Our goal is to estimate, in a distributed, real-time fashion, the mean of a dynamic global quantity of interest in a multi-robot system, while still using fixed-size messages. The simple approach of broadcasting the local state of each agent will eventually use all available communication bandwidth as population increases. Consensus al-

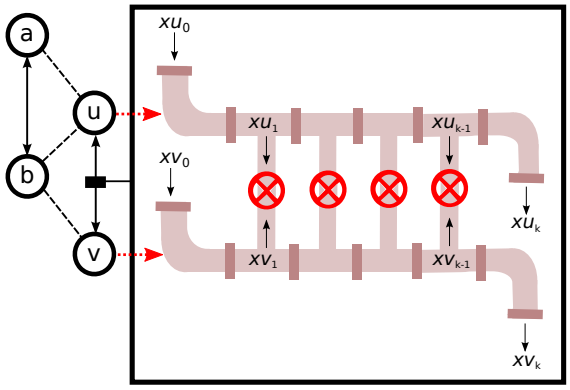


Figure 1: A network of agents performing pipelined consensus, an extension to pair-wise gossip-based consensus. Each node stores k values in their pipeline, which is a queue containing multiple concurrent consensuses. In this example, agents u and v are in the middle of gossiping. At the beginning of a gossip, a new value is inserted into the pipeline, and older values are shifted along the pipeline. The two agents then perform consensus on each cell of their pipelines. The oldest value in the pipeline is taken as the current estimate of the global value. Pipelined consensus is a robust practical solution to global state estimation problems in multi-agent systems.

Appears in: *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum (eds.), May, 4–8, 2015, Istanbul, Turkey.*

gorithms can compute global estimates using fixed-size messages between agents, but are not robust to real-world errors, with a few exceptions [20]. Real-world networks are subject to many other types of errors. These include changes in the network's structure due to the dynamic nature of wireless communications, topology changes, communication errors, and sensor errors will cause normal consensus algorithms to have a poor estimate of the global value [10].

In this paper, we present the *pipelined consensus* algorithm. It differs from existing standard consensus algorithms in that it can continuously track a changing global mean while being robust to disturbances such as poor initial conditions, topology changes, sensor errors, and communication failures. It strikes a balance between communication cost and robustness to allow for effective operation in real-world systems. These properties make pipelined consensus highly applicable to multi-agent applications subject to the complications of the real-world.

The core idea behind our algorithm is that the agents start a new consensus each round; initializing them with new measurements of local state. Older consensus estimations are stored in a pipeline queue in order. Consensus estimations corresponding to similar start times will evolve together in every communication round, largely independent of estimations belonging to different start times, as shown in Figure 1. A pipeline size, k, must be chosen such that the estimation will approach the true average before the pipeline runs out of space. This is the fundamental trade-off of our approach: the pipeline size, k, must be larger than the natural convergence time of consensus in the network, τ. Larger values for k allow us to handle larger, more complex networks, but use more communication bandwidth.

One important aspect of tracking a changing consensus is the rate of convergence. [1, 4, 16, 17], which derived upper bound on the convergence time under different assumptions and from different prospective. In this paper, we study the convergence time τ for a large set of mesh networks, and demonstrate that pipelined consensus is a feasible solution for the vast majority of them. We then present two examples and many experiments to demonstrate the robustness properties of the algorithm : computing the global mean of a quantity in a distributed sensor network, and estimating the centroid of a group of robots.

The rest of the paper is organized as follows. Related works are described in Section 2. We describe our model and assumptions in Section 3. The pipelined consensus is formally proposed in Section 4. Extensive simulation and experiment results are provided in Section 5, where we use pipelined consensus to track a changing average and perform centroid estimation. Convergence time and tracking error subject to different disturbances are also analyzed. We discuss and conclude our work in Section 6 and 7.

2. RELATED WORK

There is a large literature regarding consensus in multi-agent systems [11, 14]. The original fundamental work, [15] proposed the a consensus algorithm where all agents in a group could agree on a common value only by communicating with their neighbors in the network. Another major approach to achieve *average consensus* is to let agents gossip in a pair-wise fashion and use the coordination law in [8]. Our work uses this basic approach, but runs multiple consensus in parallel. Many applications of consensus use a value that could be constantly changing, such as an environmental sensor or other controlled signals. To handle this, the concept of dynamic consensus was introduced in [21, 22] and convergence was proved. More generally, [3, 25] used a proportional-integral filter on the typical consensus algorithm to achieve robust average tracking of time-varying inputs regardless of the initial states. In both dynamic and robust consensus, the input signals to the nodes must be known, whereas our pipelined consensus algorithm does not have this requirement. We apply our pipelined consensus algorithm to estimate the centroid of an object. There are similar work that use only local reference frames to esitamte the centroid [2, 5]. In Aragues et al work Aragues [2], each agent was assumed to get the estimate from all its neighbor and processes them in one round. Franceschelli and Gasparri [5] used a gossip-based consensus to estimate the centroid. Their consensus algorithm handles the noisy sensor error; however, the message loss was not addressed in their work.

Communications failure or message loss is difficult to handle for most consensus algorithms. The work in [20], injects a portion of the input signal into the state estimate of each robot during each round. This lets the estimated global mean on each agent be robust to communication errors, but at the cost of a large variance across individual estimates. Our approach produces qualitatively similar results, but without the additional variance caused by re-injecting the sensor value into the local estimate.

3. MODEL AND ASSUMPTIONS

We assume that our agents are in an environment too large for centralized communication. A communication network is built by the agents using inter-agent communications between nearby agents within a fixed distance d, where d is much smaller than the size of the environment. Each agent constitutes a vertex $u \in V$, where V is the set of all agents and E is the set of all agent-to-agent communication links. n is $|V|$, the total number of agents in the network. We model the agent's communications network, $G = (V, E)$, as an undirected unit disk graph. We assume that G is connected. The neighbors of each vertex u are the set of agents within line-of-sight communication range d of agent u, denoted by $\mathcal{N}(u) = \{v \in V \mid \{u, v\} \in E\}$.

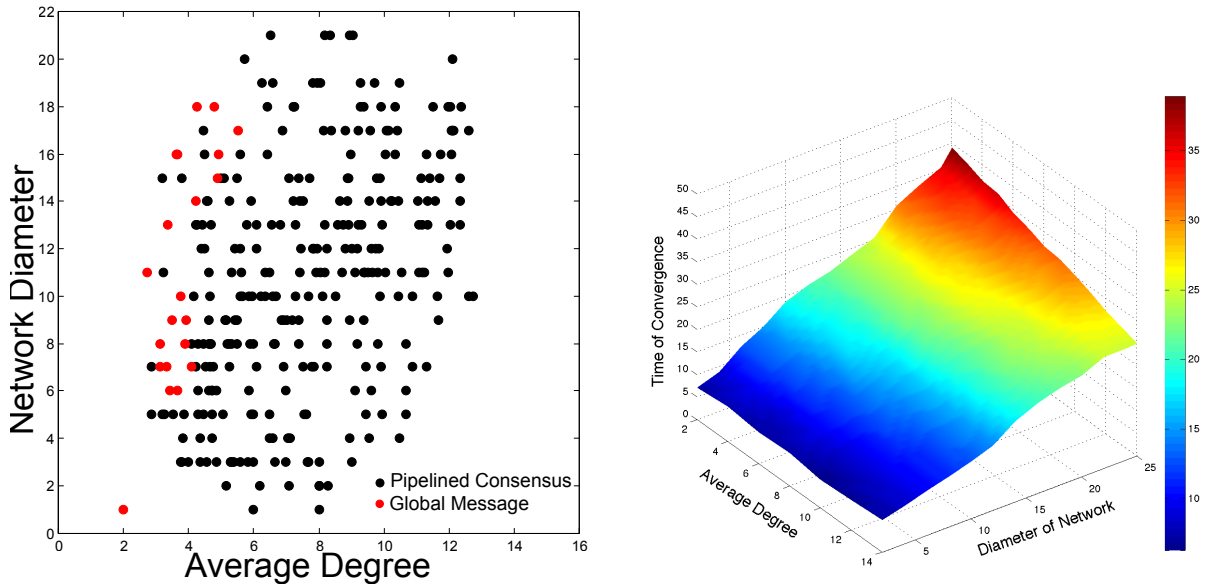
We model algorithm execution as proceeding in a series of discrete *rounds*. While actual operation in many practical systems is asynchronous, implementing a synchronizer simplifies analysis greatly and is easy to implement [12]. In this paper, we focus on the convergence time of consensus τ measured by the number of rounds of computation required to achieve some ε-bound on error.

We assume our agents are homogeneous and are modeled as a disk. Each agent is situated at the origin of its own local coordinate frame with the $\hat{x}$-axis aligned with its current heading. Agents can measure the relative pose of its neighbors. The relative pose between two agents u and v is given by three measurements, bearing B_{uv}, orientation O_{uv}, and range R_{uv}. Bearing is the angle from agent u's heading to agent v's relative position. Orientation is the angle of agent v's heading from B_{uv}. Range is the distance between agent u and v.

4. PIPELINED CONSENSUS

4.1 The Algorithm

Pipelined consensus is based upon gossip consensus, in which agents perform pairwise averages with neighboring agents. In pipelined consensus, each agent stores k values instead of only one value (See Figure 1). Pipelined consensus is described in Algorithm 1. At first, every agent's pipeline, P_i, is initialized with null values (Line1). Pipeline values are updated each round of successful pair-wise gossip with a neighbor. During a gossip, each agent first checks the size of non-null cells in the pipeline, $|P_i|$. If the pipeline is filled with values, the oldest value is removed from the



(a) **(b)**

Figure 2: **(a)** Viability of algorithms under networks of different degrees and diameters. 300 networks were sampled for 20 trials each. The red dots show a region where a global message is preferred, while the black dots show where pipelined consensus is preferred. Algorithm preference was determined by message size. If the pipeline's size k was greater or equal to n, the size of the network, a global message was preferred. Otherwise, pipelined consensus was preferred, because it needs the smaller message size. The majority of results showed that pipelined consensus was preferred. **(b)** Plot of convergence time for 10% error as a variable of diameter and degree of a network. Convergence time τ is in number of rounds. The same networks as in **(a)** were used, 20 trials each. The time of convergence is strongly related to the connectivity of the network. Networks with a large average degree have a smaller τ even in the network with high dimension. However, the diameter in the network with the lower degree affects on the time of convergence, such that the network with low degree and high dimension have the larger τ.

Algorithm 1 Pipelined Consensus

1: $P_i \leftarrow$ **NULL** ▷ Initialize all pipeline cells with null values
2: **Repeat forever on each robot** u
3: Randomly choose a neighbor $v \in \mathcal{N}(u)$ to gossip
4: **if** $|P_u| = k$ **then** ▷ $|P_u|$ is the number of non-null cells
5: REMOVE(P_u) ▷ Remove the oldest value
6: **end if**
7: INSERT(P_u, xu_0) ▷ Insert new input value
8: **for** $t = 0 :$ MIN($|P_u|, |P_v|$) **do** ▷ Perform consensus on pipeline
9: $xu_t =$ CONSENSUS(xu_t, xv_t)
10: **end for**

tail of the pipeline (Lines 4-7). This keeps the size of the pipeline constant at k, as values are always inserted. The current input value, xi_0, is inserted to the head of the pipeline. Next, consensus is performed on each respective cell in the pipelines.

Note in Line 8 of Algorithm 1, we take the minimum of the size of the two pipelines. Consensus is only performed on values that can be paired between the two pipelines, bounded by the minimum size of the pipelines.This is done because equal pipeline sizes are not guaranteed. A feature of pair-wise gossip algorithms is that for each round, every agent may or may not preform a consensus. As values are only inserted upon successful pairings and consensuses, the pipeline sizes are not always equal. This is done to be more efficient with our sampling and message size. Inserting a new value every round where consensus may not occur would dramatically increase the size of k. By accepting different sizes of pipelines, our message size stays reasonable. Disadvantageous network configurations and poor random choices can lead an agent not to perform consensus for a long time. However, for applications that sample values and errors from time independent distributions, the effects of this desynchronization are irrelevant. The reason we chose to only perform consensus on the beginning cells of the pipeline is that those values will have gone through the same number of rounds of consensus, and are therefore more temporally similar.

4.2 Convergence Time

Convergence time and other properties of a consensus algorithm are well understood to be related to the connectivity matrix of a network [15]. Since our agents are assumed to only have a local knowledge about the network topology, they cannot find the exact time of convergence. As our algorithm is an extension to consensus algorithms, it has the same properties. Therefore, we cannot converge faster than the linear consensus algorithm, but we can handle different errors that are more relevant to our applications in the real world, as we show in the experiment section.

Selecting an appropriate pipeline size k is key in piplelined consensus. If k is not large enough, the final estimate from the pipeline for any round will not have sufficiently converged. This gives a larger error and variance in the estimate. Based on the convergence time of the network τ, we select a pipeline size k to to reach a desired amount of convergence. However, large values of k have a large message size which is a limiting factor on real communication systems with limited bandwidth. In order to find a balance between

τ and k, we analyze the convergence time of different networks to understand τ and k more thoroughly. Figure 2b shows the time of convergence for the same sample of networks used in Figure 2a. This result illustrates the relation between the degree and diameter of the network, and the convergence time of the network.

In pipelined consensus, each agent shares the contents of its pipeline with its neighbors, which is a message of size k, the pipeline size. If k is equal to or greater than n, *i.e.* the number of agents in the network, then there is no advantage in using the pipelined consensus algorithm. In cases such as this, agents could use a *Global Message* to share its estimate with other agents. This message would consist of the value from each agent network, from which the global value could be obtained. We tested the pipelined consensus algorithm on a large sample of networks of varying degree and diameter to show the viability of the algorithm as a solution to global state estimation. As Figure 2a shows, pipelined consensus is more practical than a global message in the majority of cases. However, the global message is more useful in networks with low degree and high diameter. This is because consensus performs poorly on networks with weak connectivity(low degree) and a large path between two agents(large diameter).

5. EXPERIMENTS

We tested pipelined consensus on both simulated and physical platforms. For our physical experiments, we used the r-one robot [13] as our platform. These robots use an infra-red communication system to communicate with their neighbors in synchronous rounds of 1500 milliseconds. The communication system also measures the relative pose to each neighboring robot (see Section 3). We tested pipelined consensus in two applications: Linear Average and Centroid Estimation. Pipelined consensus is also highly robust to change in values, sensor errors, population changes, and communication failures. All of these are a feature of real applications and can be handled by our algorithm. We show how our algorithm reacts to these effects in both simulations and physical experiments.

5.1 Global Linear Average

5.1.1 Physical Experiment, Ideal Conditions

We validated the pipelined consensus algorithm on real robotic agents. We distributed 24 agents on the floor in a grid shape of 4 by 6, which creates a network with the average degree of 5 and a diameter of 5. We used average consensus for the gossip protocol in the pipeline. Each agent starts estimation from some constant random initial value. This experiment was carried out under the most ideal conditions possible on the physical platform. However, sensor errors and communication failures inherent to physical systems still occurred. Figure 3a shows the estimation progress in each agent in an example experiment. The pipeline size k for this experiment is 20. The result shows how pipelined consensus produces estimates on all robots that have a mean error of approximately zero, but with some variance.

We ran 6 trials of this experiment. Figure 3b summarizes the result in tracking the mean of the error estimate with deviations over these trials.

5.1.2 Simulation, Communication Failure

In pairwise average consensus, a single measurement is taken and used by the agents to come to agreement. This means that any communication failure that occurs can lead to a large divergences in the estimated value. Pipelined consensus solves this problem by constantly inputting and pushing values out of the pipeline, so that values that contain error are flushed out after a maximum k rounds.

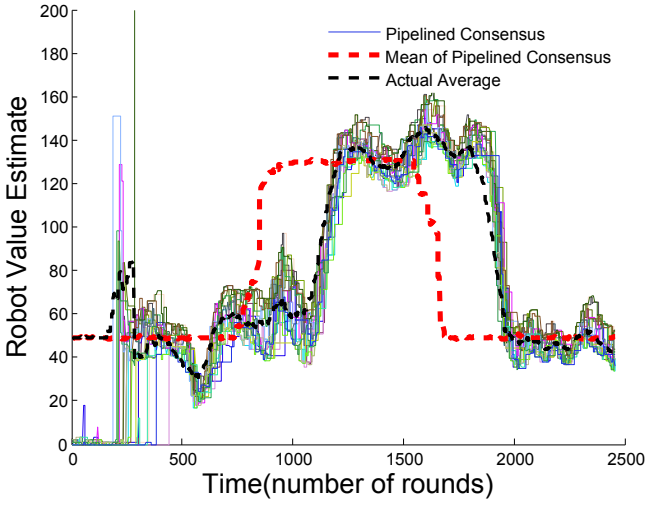


Figure 5: Light tracking experiment by 20 robots. The read sensor value was averaged together. The light began off for 20 minutes, was then turned on for 20 minutes, and then back off for the final 20 minutes. Note the delay of a factor of k rounds in the read input signal to the estimated value. The red dashed line shows the actual average of light measurement on 20 robots. The black dashed line shows the average of reported values from the consensus on robots. Solid lines show each robot's estimation. One robot had a large bias in reading the light intensity. As a feature of consensus, this value washed out after the consensus occurred.

Figure 4a shows how communication errors effect both pipelined consensus and average consensus. Average consensus converges to a value with very small variance but with an offset from the actual global mean. On the other hand, pipelined consensus continually estimates values with some variance around the global mean with an average error of approximately zero. The variance demonstrated by pipelined consensus can be decreased by increasing the size of k. Pipelined consensus can tolerate communication failure, but an increase in communication failure requires a larger time to converge to an adequate value. That requires pipeline size increases as communication failure increases. Figure 4b shows the effects of increasing communication error to the size of a pipeline for different error tolerance.

5.1.3 Physical Experiment, Changing Value

As pipelined consensus is robust to changing values, it can track a changing input signal. This feature can be used to calculate the average of a sensor value across a collection of nodes spread out in an environment, such as temperature or light. We tested this feature by having our robots measure the ambient light over time. We changed the intensity of the light by turning on and off the overhead lights in our lab in 20 minute intervals. Figure 5 shows the result of light signal tracking by 20 robots. This figure illustrates the ability to successfully track a changing input signal with accuracy. It also shows the algorithms tolerance to sensor errors. In this experiment, one robot had a very large bias in the light sensor value. However, the converging values still reached consensus over the network.

5.2 Centroid Estimation

In multi-agent manipulation tasks, mobile agents require knowledge of the geometry of the object they are transporting in order to properly manipulate the object [7]. The centroid is a value of the object that can be estimated by agents using only their own posi-

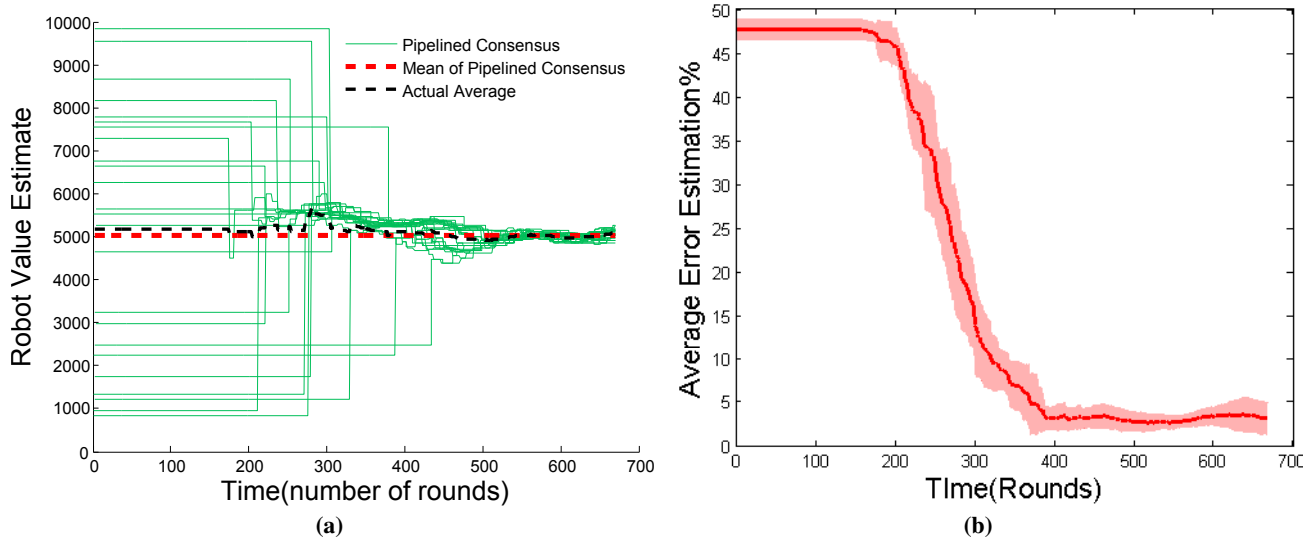


Figure 3: **(a)** Experiment result of pipeline consensus for a network with 24 agents to estimate the linear average of global values. The average degree of network is 5 and its diameter is 5. Agents only give their estimate after the pipeline has been fully filled, which gives the leading edges of their input values at the start of the experiment. The average error of their estimate is 2.07% in this experiment. **(b)** The result of pipelined consensus for 6 trials of the same experiment in **(a)**. We take the average error estimate in each trial. The solid red line shows the average of the average errors over 6 trials. The standard deviation of the average error is shown by the shaded red area. This result shows that the error decreases to 2% with pipelined consensus with small variance.

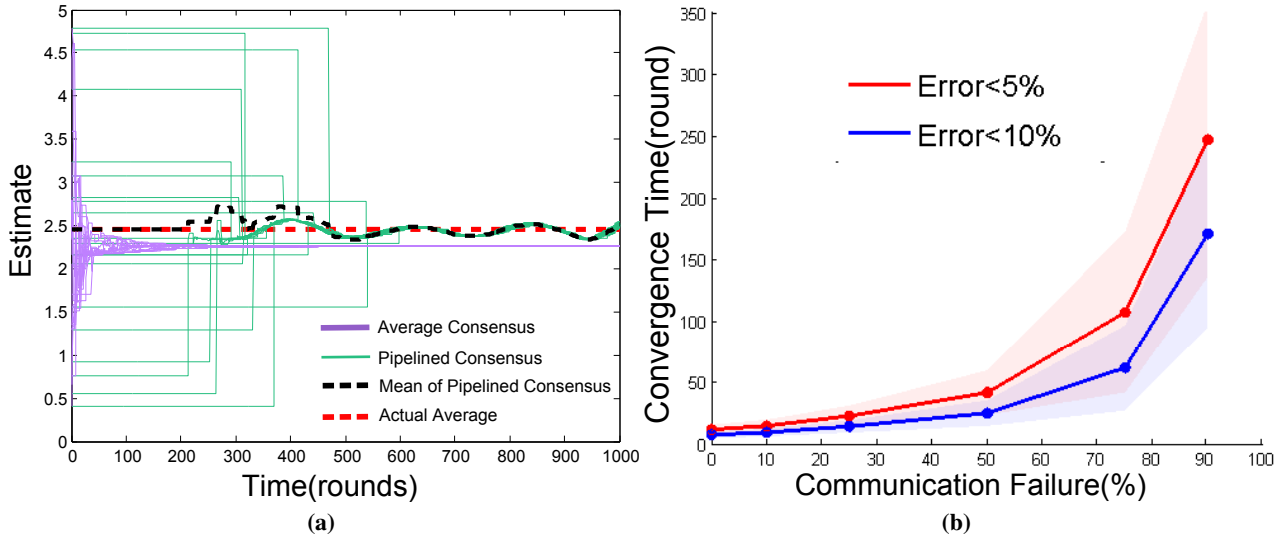


Figure 4: **(a)** Comparison of pipelined consensus and average consensus in a network of 20 agents in computing linear average. The communication failure is 75%. Agents initially have random value between 0 to 5. The size of k in this experiment was 100. The communication errors cause the variance of the result from the pipelined consensus to increase, but the estimated mean remains stable around the actual mean. Average consensus converges to an erroneous value and remains. **(b)** The effect of communication failure in the range of 0% to 90% on the time of convergence with error in the consensus estimate less than 10% (blue solid line) and less than 5% (red solid line). The network is of 72 agents. This result shows the average and standard deviation of the convergence time over 12 trails.

tions. We assume that the robots are distributed around the object in such a configuration that they approximate the shape of the object. In this case, the centroid can be approximated by taking the average of the positions of all the robots. Simulations and hardware experiments in this section show that the centroid can be found even without a global reference frame by using pipelined consensus.

We assume there is no global reference frame, which means robots estimate in their local reference frames. Therefore, the resulting consensus values are all different for each robot, although they correspond to a common point in the global frame. However, it is impossible for the robots to actually know this in a distributed system. In order to communicate values from one robot to

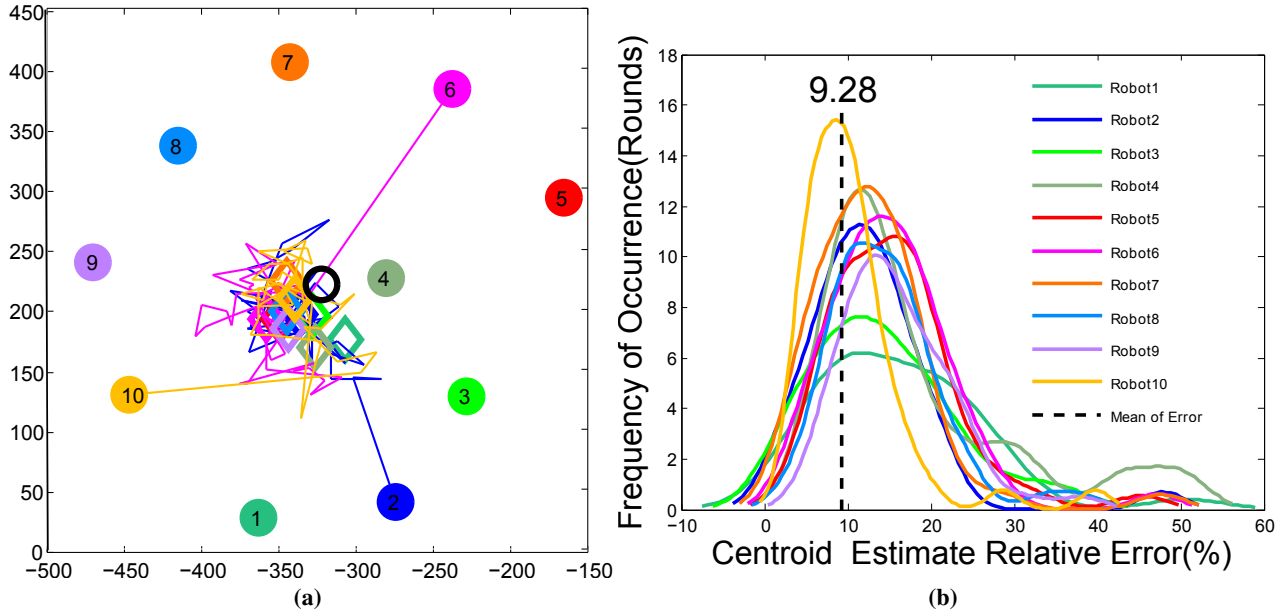


Figure 6: **(a)** Physical Experiment: centroid estimation by 10 robots in a concave shape configuration. The robots positions are designated by the solid circles numerically labeled. Three of robot's estimate over time is shown by the solid lines of similar color emanating from the circle. Each robots average estimate is shown by the colored diamonds. The true centroid is shown by the black circle. The average error is 9.28% for this experiment. **(b)** We fit Kernel density model to the distribution of centroid estimation error for each robot in the experiment illustrated in (a). The colors match with ones in the illustration(a). The mean error is 9.28% and is shown by dashed black line.

another, a coordinate transformation between different local reference frames must be performed. Consider a point x in the global reference frame. Its coordinates in robot u's and v's local reference frame are denoted by ${}^{u}x$ and ${}^{v}x$ respectively. The relationship between ${}^{u}x$ and ${}^{v}x$ is given by:

$$ {}^{u}x = {}^{u}_{v}M\,{}^{v}x, \tag{1} $$

where

$$ {}^{u}_{v}M = \begin{bmatrix} \cos(\theta_{vu}) & -\sin(\theta_{vu}) & d_{vu}\cos(B_{vu}) \\ \sin(\theta_{vu}) & \cos(\theta_{vu}) & d_{vu}\sin(B_{vu}) \\ 0 & 0 & 1 \end{bmatrix} $$

is a coordinate transformation matrix, and

$$ \theta_{vu} = \pi - O_{vu} + B_{vu}. $$

In the equations above, B_{vu} and O_{vu} are the bearing angle and orientation angle respectively and d_{vu} is the distance between agent u and v measured by agent u [13]. All these angles and distance can be measured locally by sensors. θ_{vu} is the relative heading of v from u.

In order to estimate the centroid, each robot maintains two pipelines, one for x coordinate and another for y coordinate of the estimated centroid. The initial input values for two pipelines are both zero, which represents the robot's position in its own reference frame. The relative poses of the neighbors are measured in every round. Robots use Algorithm 1 to update pipelined consensus in each round. However, the robots transform their neighbor's pipeline values by using the aforementioned coordinate transformation. The value in the last cell of the x and y pipeline is considered as the current estimation for the centroid.

5.2.1 *Physical Experiment, Ideal Conditions*

We tested pipelined centroid estimation on real agents. We used the r-one robot as our robotic agents. 10 robots were used in a configuration illustrated in Figure 6a. This figure shows the robot's estimation of the centroid over the time. In this experiment, the average error of the estimate was 9%. The distribution of error for the centroid estimate is approximated by the kernel density estimation in Figure 6b. In this experiment we used the AprilTag system [18]to measure distance between robots, eliminating error resulting from poor distance measurements on the robots. However, the error resulting from poor angular pose estimation using the infrared sensors still influenced the result. The resolution of the bearing and orientation is limited to 22.5° slices [13]. This result shows that the centroid is accurately estimated by pipelined consensus in the presence of sensor error.

5.2.2 *Simulated Experiment, Sensor Error*

In centroid estimation, sensor error is introduced by using the coordinate transformation. The angular and distance measurements sampled by the agents could be very noisy. In an average consensus, the estimate value evolves based upon a single sensor measurement or value taken at the beginning. This value has some unknown error from the sensor measurement that is never removed from the estimate. Our pipelined consensus algorithm continually re-samples the state of the network and sensors, and thus reduces error in estimation. In Figure 7, we demonstrate how the variance of the centroid estimation is related to the variance of the sensor measurements in a randomly generated unit disc graph. The distance and angle measurements are modeled by zero-mean Gaussian model. As we can see in the figure, the variance of the centroid estimation is almost linear to the variance of the sensor, and the angle measurement has a larger impact on the accuracy of the estimation. This is due to the coordinate transform, as angular errors will result in points moving a much greater distance from their actual positions than errors in the distance. Figure 8 provides a comparison between centroid estimation using average consensus and pipelined consensus. Average consensus has a larger estimation error, despite be-

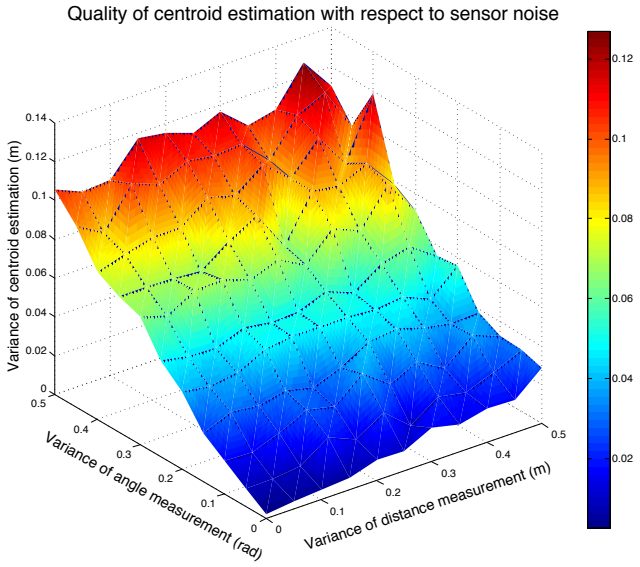


Figure 7: Simulation of centroid estimation with 20 robots. The graph is a randomly generated unit disc graph. The average degree is 7 and the network diameter is 3. We model the noisy sensor using zero-mean Gaussian model. The 3D plot here demonstrates the change of average variance of the centroid estimations of all robots as we increase variance of angle and distance measurements from 0 to 0.5.

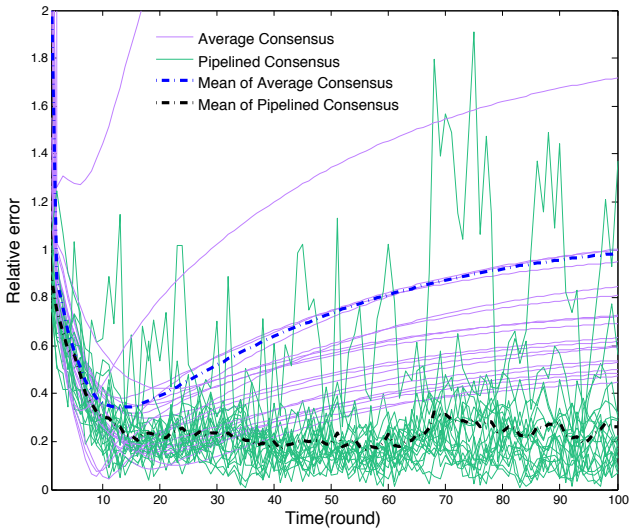


Figure 8: Comparison of average consensus and pipelined consensus over time on the same network as Figure 7. The y-axis shows the relative error, defined as the distance between the true and the estimated centroid, divided by the distance between the robot and the true centroid. Relative errors of all the robots using either pipelined or average consensus are plotted. The variances of both the distance and angle error in sensor reading are set to 0.1.

ing much smoother than the values from pipelined consensus. Our pipelined consensus approach gives a much lower mean of relative error, but also has greater variance than the average consensus. Note that the average consensus does not reach a common value in this case because the sensor reading $\theta_{vu} \neq -\theta_{uv}$, $d_{vu} \neq d_{uv}$ due to noise. This inconsistency, when robots exchange their estimations using coordinate transformation, causes the estimations of different robots to diverge.

5.2.3 *Physical Experiment, Population Changes*

Another feature of pipelined consensus is that it is self-stabilizing in regards to changing population and topology. We examined the effect of population changes on a network of robots performing centroid estimation. We began the test with 5 robots, and added and subtracted robots from the population in different areas of the network over time. Figure 9 shows the error in the estimate over time with changes in population. We added and removed robots only after the estimate has stabilized to a sufficient degree. Results show the error in the estimation remains around 9% for all population sizes and changes.

6. DISCUSSION AND LIMITATIONS

Consensus algorithms are anytime algorithms, however the amount of variance in the value is dependent on the time it has been running. The value k that we introduce can be changed to determine the amount of variance that is desired in the final value of the pipeline. As the pipeline contains a discrete amount of values, it cannot be left to run indefinitely and therefore has to have a bounded variance. With an understanding of the network, we can determine k suitable to a level of variance that is acceptable. In real applications, robots themselves cannot make this decision. This would require global knowledge of the network, which they cannot know from a distributed perspective.

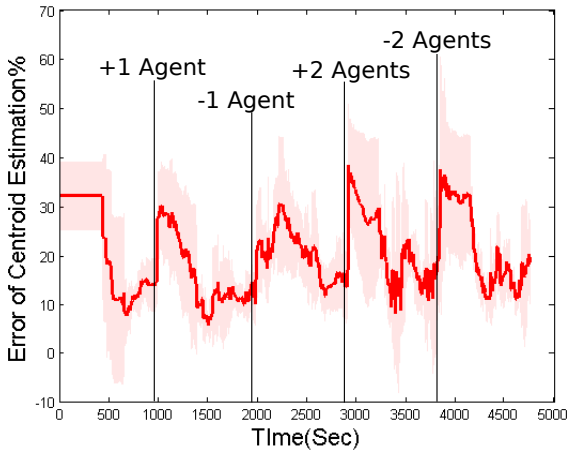


Figure 9: Physical experiment: illustration of robustness of pipelined consensus to population changes during the centroid estimation. Robots were added and removed over time after estimate stabilization. The robots removed and added were in different physical areas of the network each time. Each time a robot is added or removed from the network, error increases as the actual centroid no longer matches the estimated centroid. Over time, the estimate reconverges to the actual centroid within an error of 9% for all populations.

Our pipeline consensus algorithm achieves dynamic average tracking with high robustness to the initial condition, sensor errors, population changes, and communication failures. There are other algorithms that do the similar job, like the PI consensus filter designed in [3]. However, each agent must know the input signal for this approach. This may not be true in all multi-agent systems as some agents may not know the input signal for themselves or can only know the input signals for their neighbors. For example, in the centroid estimation task we described above, the only thing robots know about themselves is their position in their reference frame, which is always $[0,0]$. There is no knowledge about the input signal in this case, so the algorithm is not applicable.

Pipelined consensus is time-sensitive in nature. Data inserted in the pipeline will only be produced from the pipeline after τ rounds of successful consensus with neighbors. However, the consensus operation is not related to time. The shifting values in pipeline consensus make it important to stay relatively synchronized. A poor configuration in the network or simple unluckiness may lead to an agent taking significantly longer time to produce an updated result. This can be seen in practice in Figure 3a. Some robots in the experiment produced an estimate much later than others, taking more time to achieve consensus τ times. This also can lead to stale data reentering the system and affecting the final result. For a system with a changing input signal, a robot who has not performed consensus for a relatively long time may perform consensus with another, more up-to-date robot. Performing consensus with the old data inserts error into the system, as the result for the temporally offset input values will be different.

The centroid estimation is sensitive to motion of the agents. This is because geometric measurements used in consensus are sensitive to the geometry of the network at the time the measurement was taken. The consensus estimate will become offset by the angular and translational motion. Since there is a lag of at least τ in pipeline consensus for information update, the error between the true centroid and the current estimation will grow the faster the agents change position. Hopefully, this error can be reduced by increasing the frequency of the communication. The estimation error caused by moving robots can be handled more efficiently by maintaining history information of pose of other agents at each stage of the pipelined consensus. This solution is left as a future work.

7. CONCLUSION

We have demonstrated that Pipelined consensus is a robust and practical extension to pairwise consensus algorithms for multi-agent systems. In all of our experiments, the algorithm handled many different types of errors well, quickly converging to accurate global estimates. In the future, we plan to rigorously study how to more accurately compute τ, which is related to features of the graph topology such as number of nodes, average degree, diameter, closeness, min-cut, etc. A mathematical characterization of τ will help ensure the effectiveness and efficiency of the pipelined consensus algorithm. Analysis of the variance introduced by communications errors can help understand the convergence rates in tough communications environments. As for the centroid estimation, we plan to integrate it into a multi-robot manipulation task and test its performance when the robots actually move with the object.

We tested pipelined consensus on the experiment in which the mean value of global state is desired. In our future work, we will show pipelined consensus can be used for any convex operation.

8. ACKNOWLEDGMENTS

This work has been supported by National Science Foundation, Division of Computer and Network Systems under CNS-1330085.

9. REFERENCES

[1] D. Angeli and P. Bliman. Convergence speed of distributed consensus and topology of the associated information spread. In *Decision and Control, 2007 46th IEEE Conference on*, pages 300–305. IEEE, 2007.

[2] R. Aragues, L. Carlone, C. Sagues, and G. Calafiore. Distributed centroid estimation from noisy relative measurements. *Systems & Control Letters*, 61(7):773–779, 2012.

[3] H. Bai, R. A. Freeman, and K. M. Lynch. Robust dynamic average consensus of time-varying inputs. In *Decision and Control (CDC), 2010 49th IEEE Conference on*, pages 3104–3109. IEEE, 2010.

[4] M. Cao, A. S. Morse, and B. D. Anderson. Reaching a consensus in a dynamically changing environment: Convergence rates, measurement delays, and asynchronous events. *SIAM Journal on Control and Optimization*, 47(2):601–623, 2008.

[5] M. Franceschelli and A. Gasparri. Gossip-based centroid and common reference frame estimation in multiagent systems. 2014.

[6] J. Ghaderi and R. Srikant. Opinion dynamics in social networks: A local interaction game with stubborn agents. In *American Control Conference (ACC), 2013*, pages 1982–1987, June 2013.

[7] G. Habibi, K. Zachary, W. Xie, M. Jellins, and J. McLurkin. Distributed centroid estimation and motion controllers for collective transport by multi-robot systems. In *Robotics and Automation (ICRA), 2015 IEEE International Conference on*. IEEE, May 2015.

[8] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *Automatic Control, IEEE Transactions on*, 48(6):988–1001, 2003.

[9] B. J. Julian, M. Angermann, M. Schwager, and D. Rus. A scalable information theoretic approach to distributed robot coordination. In *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, pages 5187–5194. IEEE, 2011.

[10] T. Li and J.-F. Zhang. Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises. *Automatic Control, IEEE Transactions on*, 55(9):2043–2057, 2010.

[11] N. A. Lynch. *Distributed Algorithms*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 1996.

[12] J. McLurkin. *Analysis and Implementation of Distributed Algorithms for Multi-Robot Systems*. PhD thesis, MIT ,USA, 2008.

[13] J. McLurkin, A. McMullen, N. Robbins, G. Habibi, A. Becker, A. Chou, H. Li, M. John, N. Okeke, J. Rykowski, S. Kim, W. Xie, T. Vaughn, Y. Zhou, J. Shen, N. Chen, Q. Kaseman, L. Langford, J. Hunt, A. Boone, and K. Koch. A robot system design for low-cost multi-robot manipulation. In *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, Chicago, IL, USA, September 14-18, 2014*, pages 912–918, 2014.

[14] R. Olfati-Saber, J. Fax, and R. Murray. Consensus and

cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1):215–233, 2007.

[15] R. Olfati-Saber and R. M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *Automatic Control, IEEE Transactions on*, 49(9):1520–1533, 2004.

[16] A. Olshevsky and J. N. Tsitsiklis. Convergence speed in distributed consensus and averaging. *SIAM Journal on Control and Optimization*, 48(1):33–55, 2009.

[17] A. Olshevsky and J. N. Tsitsiklis. Degree fluctuations and the convergence time of consensus algorithms. *Automatic Control, IEEE Transactions on*, 58(10):2626–2631, 2013.

[18] E. Olson. AprilTag: A robust and flexible visual fiducial system. In *Proceedings of the IEEE International Conference on Robotics and Automation (ICRA)*, pages 3400–3407. IEEE, May 2011.

[19] M. Schwager, D. Rus, and J.-J. Slotine. Decentralized, adaptive coverage control for networked robots. *The International Journal of Robotics Research*, 28(3):357–375, 2009.

[20] F. Shaw, A. Chiu, and J. McLurkin. Agreement on stochastic multi-robot systems with communication failures. In *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, pages 6095–6100, Oct 2010.

[21] D. P. Spanos, R. Olfati-Saber, and R. M. Murray. Distributed sensor fusion using dynamic consensus. In *IFAC World Congress*, 2005.

[22] D. P. Spanos, R. Olfati-Saber, and R. M. Murray. Dynamic consensus on mobile networks. In *IFAC world congress*. Prague Czech Republic, 2005.

[23] H. G. Tanner, A. Jadbabaie, and G. J. Pappas. Flocking in fixed and switching networks. *Automatic Control, IEEE Transactions on*, 52(5):863–868, 2007.

[24] Z. Wang and M. Schwager. Multi-robot manipulation without communication. In *Proc. of the International Symposium on Distributed Robotic Systems (DARS 14)*, November 2014.

[25] P. Yang, R. A. Freeman, and K. M. Lynch. Distributed cooperative active sensing using consensus filters. In *Robotics and Automation, 2007 IEEE International Conference on*, pages 405–410. IEEE, 2007.